



AS Mathematics MPC1

Unit: Pure Core 1

Mark scheme

June 2017

Version: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
-x EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

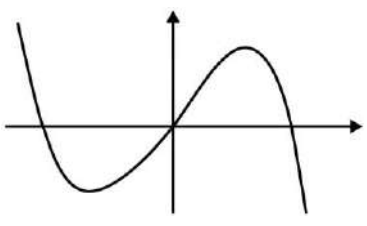
Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, the principal examiner may suggest that we award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q1	Solution	Mark	Total	Comment
	NO MISREADS ALLOWED IN THIS QUESTION			
(a)	$\frac{1+4\sqrt{7}}{5+2\sqrt{7}} \times \frac{5-2\sqrt{7}}{5-2\sqrt{7}}$ <p>(Numerator =) $5+20\sqrt{7}-2\sqrt{7}-56$</p> <p>(Denominator = $25+10\sqrt{7}-10\sqrt{7}-28$) $= -3$</p> <p>Value = $\frac{-51+18\sqrt{7}}{-3}$ $= 17-6\sqrt{7}$</p>	<p>M1</p> <p>A1</p> <p>B1</p> <p>A1cso</p>	<p>4</p>	<p>at least this far</p> <p>must be seen as denominator</p> <p>condone $-6\sqrt{7}+17$</p>
(b)	$x(9\sqrt{5} - \text{"their"}6\sqrt{5}) = \text{"their"}4\sqrt{5}$ $x(9\sqrt{5} - 6\sqrt{5}) = 4\sqrt{5}$ $x = \frac{4}{3} \quad \text{or} \quad x = 1\frac{1}{3} \quad \text{or} \quad x = 1.\dot{3}$	<p>M1</p> <p>A1</p> <p>A1</p>	<p>3</p>	<p>attempt to write each term as $k\sqrt{5}$ with either $2\sqrt{45} = 6\sqrt{5}$ or $\sqrt{80} = 4\sqrt{5}$</p> <p>OE must have equation</p> <p>must be simplified to one of these</p>
	Total		7	
(a)	<p>Condone multiplication by $5-2\sqrt{7}$ instead of $\times \frac{5-2\sqrt{7}}{5-2\sqrt{7}}$ for M1 only if subsequent working shows multiplication by both numerator and denominator – otherwise M0</p> <p>For first A1 each term must be evaluated correctly but may be seen in a grid</p> <p>An error in the denominator such as $25+5\sqrt{7}-5\sqrt{7}-28=-3$ should be given B0 and it would then automatically lose the final A1cso</p> <p>May use $\frac{1+4\sqrt{7}}{5+2\sqrt{7}} \times \frac{2\sqrt{7}-5}{2\sqrt{7}-5}$ M1; numerator = $-5-20\sqrt{7}+2\sqrt{7}+56$ A1 etc</p> <p>Alternative: $\frac{1+4\sqrt{7}}{5+2\sqrt{7}} = m+n\sqrt{7}$</p> <p>leading to $5m+14n=1$; $2m+5n=4$ M1 (one correct) A1(both correct)</p> <p>either $m=17$ or $n=-6$ A1; answer = $17-6\sqrt{7}$ A1 cso (expression must be explicitly seen)</p>			
(b)	<p>Alternative 1 Multiply or divide each term by $\sqrt{5}$ obtaining 3 integer terms (possibly with one error) M1</p> <p>3 correct integer terms; eg $x(45-30) = 20$ A1 ; $x = \frac{4}{3}$ A1</p> <p>Alternative 2 $x = \frac{\sqrt{80}}{9\sqrt{5}-2\sqrt{45}} \times \frac{9\sqrt{5}+2\sqrt{45}}{9\sqrt{5}+2\sqrt{45}}$ M1 = $\frac{300}{225}$ OE A1(integer/integer) = $\frac{4}{3}$ A1</p> <p>Squaring: Do not allow final A1 unless justification for rejecting negative value</p> <p>May earn M1 for $x^2(k\sqrt{5})^2 = 80$ and A1 for $x^2 = \frac{16}{9}$ OE</p>			

Q2	Solution	Mark	Total	Comment
(a)	$\left(\frac{dy}{dx} =\right) 20 - 2x - 6x^2$ $(10 - 6x)(2 + x) (=0) \quad \text{OE}$ <p>(other stationary point when) $x = \frac{5}{3}$</p>	M1 A1 A1 A1cso	 4	two terms correct all correct correct factors or correct use of formula for correct quadratic possibly multiplied by -1 or divided by ± 2 OE eg $\frac{20}{12}$, $1\frac{2}{3}$, 1.6 but not $\frac{-20}{-12}$
(b)	$\left(\frac{d^2y}{dx^2} =\right) -2 - 12x$ $\left(\text{when } x = -2, \frac{d^2y}{dx^2}\right) = (-2 + 24) = 22$ <p>$22 > 0$ therefore minimum (point)</p>	 B1 B1ft E1ft	 3	Sub $x = -2$ into their $\frac{d^2y}{dx^2}$ and evaluate correctly FT their value of $\frac{d^2y}{dx^2}$ but must have reason
(c)	Cubic graph through origin with one max & one min on either side of y-axis 	M1 A1	 2	may be reflection of given graph in x -axis for M1 Graph roughly as shown in all 4 quadrants
Total			9	
(a)	If candidate multiplies by -1 before differentiating and writes $\frac{dy}{dx} = 6x^2 + 2x - 20$ this scores M0 For second A1 , may have $(5 - 3x)(4 + 2x)$ or $2(3x - 5)(x + 2)$ or $(5 - 3x)(2 + x)$ etc Second A1 is earned for formula as far as $\frac{2 \pm 22}{-12}$ or $\frac{-1 \pm 11}{6}$ etc If both values given $x = -2, \frac{5}{3}$ then allow A1cso Withhold A1cso if no “=0” seen or incorrect equating of expressions.			
(b)	Allow E1ft for “their $-2 + 24 > 0$ so min” etc or $\frac{d^2y}{dx^2} > 0$ (provided they have a value earlier) \Rightarrow minimum etc			
(c)	Allow M1 if the curve does not actually cross the x -axis 3 times For A1 ignore any “numbers” on graph			

Q3	Solution	Mark	Total	Comment
(a)	$(-2)^3 + b(-2)^2 + c(-2) + 24$	M1	2	clear attempt at $p(-2)$ AG must see powers of -2 simplified correctly and $= 0$ appearing before last line
	$-8 + 4b - 2c + 24 = 0$	A1		
	$2b - c + 8 = 0$			
(b)	$3^3 + 3^2b + 3c + 24 = -30$	M1	2	clear attempt at $p(3)$ and $= -30$ ACF terms need not be collected but powers of 3 must be evaluated No ISW - mark their final equation
	$27 + 9b + 3c + 24 = -30$	A1		
	$3b + c + 27 = 0$			
(c)	Correctly eliminating b or c from $2b - c + 8 = 0$ and an equation from (b)	M1	3	PI by one correct answer
	$b = -7$ or $c = -6$	A1		
	$b = -7$ and $c = -6$	A1		
Total			7	
(a)	Condone poor use of brackets if recovered on next line for both M1 and A1 Note that “ $= 0$ ” must appear before last line; Example $p(-2) = -8 + 4b - 2c + 24 \Rightarrow 2b - c + 8 = 0$ scores M1 A0 M1 may also be earned for a full long division attempt by $(x+2)$ as far as the remainder in terms of b and c . M1 also for $p(x) = (x+2)(x^2 + dx + 12)$ $b = 2 + d$; $c = 2d + 12$ with A1 for completion. Terms must be exactly as printed answer but accept $0 = 2b - c + 8$ for A1 .			
(b)	Do NOT treat use of 30 instead of -30 as a misread. M1 may also be earned for a full long division attempt by $(x-3)$ as far as the remainder and equating the remainder (in terms of b and c) to -30 . Ignore trailing equals sign for A1 in both parts (a) and (b).			

Q4	Solution	Mark	Total	Comment			
(a)	$\text{grad } AB = \frac{-6-5}{8--2} \quad \text{or} \quad \frac{5--6}{-2-8}$	M1	4	correct unsimplified; PI by $-\frac{11}{10}$ ft “their gradient” but must use A or B coordinates correctly integer coefficients with x and y terms on one side and integer on other side			
	$= -\frac{11}{10}$	A1					
	$y-5 = \text{“their”} -\frac{11}{10}(x--2)$	M1					
	$\text{or } y--6 = \text{“their”} -\frac{11}{10}(x-8)$						
	$11x+10y=28$	A1					
	(b)	$(\text{grad } AC =) \frac{k-4}{k+2} \quad \text{or} \quad (\text{grad } BC =) \frac{k+7}{k-8}$			B1	5	correct simplified expression eg $\frac{4-k}{-2-k}$ condone one error in one term <i>attempt</i> at factorisation or correct use of formula for “their” quadratic
		$\frac{k-4}{k+2} \times \frac{k+7}{k-8} = -1 \quad \text{OE}$			M1		
		$k^2+3k-28 = -k^2+6k+16$			A1		
		$\Rightarrow 2k^2-3k-44 (=0)$					
		$(k+4)(2k-11) (=0)$			dM1		
$k = -4, \quad k = 5\frac{1}{2} \quad \text{OE}$		A1					
Total			9				
(a)	May earn second M1 for $y = \text{“their”} -\frac{11}{10}x + c$ and attempt to find c using $x = -2, y = 5$ or $x = 8, y = -6$ For A1 , condone any equivalent equation of the correct form such as $28 = 10y + 11x$ or $-22x - 20y = -56$ but not $11x + 10y - 28 = 0$ or any equation with non-integer coefficients						
(b)	Alternative $[AC^2 =](k-4)^2 + (k+2)^2$ or $[BC^2 =](k-8)^2 + (k+7)^2$ B1 (may be under square root) Pythagoras $(k-4)^2 + (k+2)^2 + (k-8)^2 + (k+7)^2 = 10^2 + 11^2$ M1 (condone one error in one term) $k^2 - 8k + 16 + k^2 + 4k + 4 + k^2 - 16k + 64 + k^2 + 14k + 49 = 221 \Rightarrow 4k^2 - 6k - 88 = 0$ A1 etc Allow dM1 for factors that would multiply out to give their k^2 and constant terms. To earn dM1 using formula then it must be correct for “their” quadratic and discriminant evaluated “correctly”; if correct quadratic used, you must see $\frac{3 \pm \sqrt{361}}{4}$ OE to award dM1						

Q5	Solution	Mark	Total	Comment
(a)	$\left(\frac{dy}{dx} = \right) 8x^3 - 9x^2$	M1	5	one term correct
		A1		$\frac{dy}{dx}$ correct
	$\left(\frac{dy}{dx} = 8(-1)^3 - 9(-1)^2\right) = -17$	dM1		correct substitution of $x = -1$ and correct evaluation for “their” $\frac{dy}{dx}$
	(Grad of normal $=$) $\frac{1}{17}$	A1ft		FT their negative reciprocal provided M1 is earned
	$y = \frac{1}{17}x + 9\frac{1}{17}$	A1		or $y = \frac{1}{17}x + \frac{154}{17}$ equation must be in this form
(b)(i)	$\frac{2}{5}x^5 - \frac{3}{4}x^4 + 4x$	M1 A1	5	two terms correct all correct (may have +c)
	$\left[\frac{2}{5} \times 2^5 - \frac{3}{4} \times 2^4 + 4 \times 2\right] -$ $\left[\frac{2}{5} \times (-1)^5 - \frac{3}{4} \times (-1)^4 + 4 \times (-1)\right]$	dM1		correct substitution of 2 and -1 to find “their” $F(2) - F(-1)$
	$\left[\frac{2 \times 32}{5} - \frac{3 \times 16}{4} + 8\right] - \left[-\frac{2}{5} - \frac{3}{4} - 4\right]$	A1		correct with powers of 2 and (-1) and minus signs handled correctly
	$= 13\frac{19}{20}$ or 13.95 or $\frac{279}{20}$	A1		correct single equivalent fraction or decimal
(ii)	Area of trapezium $= \frac{1}{2}(9+12) \times 3$ $= 31.5$ or $\frac{63}{2}$ OE “their” trapezium – answer from (b)(i) (Area of shaded region $=$) $31.5 - 13.95$ $= 17.55$	B1 M1 A1		3
Total			13	
(a)	Do not accept $y = \frac{1}{17}x + c, \dots$ $c = \frac{154}{17}$ for final A1 ; equation must be written in full on one line.			
	Allow $y = \frac{x}{17} + \frac{154}{17}$ for final A1 but not $y = \frac{x+154}{17}$.			
(b)(i)	Use of $F(-1) - F(2)$ scores dM0			
(ii)	For M1 condone use of their “(b)(i) – “their” trapezium”. Be generous in awarding this M1 provided you are convinced they are considering the area of a trapezium.			

Q6	Solution	Mark	Total	Comment	
(a)	$(x+10)^2 + (y-7)^2 = \dots$	M1	3	one of these terms correct	
		A1		LHS correct ignoring any extra constants	
	$(x+10)^2 + (y-7)^2 = 10^2$ (or $\dots=100$)	A1		or $(x--10)^2 + (y-7)^2 = \dots$ or $(x--10)^2 + (y-7)^2 = 10^2$ (or $\dots=100$)	
	(b)	$10^2 + (y-7)^2 = 10^2$	M1	4	putting $x=0$ in “their” equation and attempt to solve for y completely correct working and both parts of the conclusion putting $y=0$ in “their” equation and attempt to solve for x completely correct working and both parts of the conclusion
		$\Rightarrow (y-7)^2 = 0 \Rightarrow y=7$	E1		
		Repeated root means circle touches y -axis	M1		
		$(x+10)^2 + 7^2 = 100$	E1		
	(c)(i)	$(x+10)^2 + (kx-5)^2 = 100$	M1	2	sub $y=kx+2$ into “their” circle equation and attempt to multiply out brackets AG be convinced - condone $0 = (1+k^2)x^2 + 10(2-k)x + 25$
		$x^2 + 20x + 100 + k^2x^2 - 10kx + 25 = 100$ $(1+k^2)x^2 + 10(2-k)x + 25 = 0$ must have terms exactly as printed answer	A1cso		
	(ii)	$10^2(2-k)^2 - 4 \times 25(1+k^2)$	M1	3	correct discriminant unsimplified multiplying out correctly must see “=0” before final answer
$400 - 400k + 100k^2 - 100 - 100k^2 (=0)$		A1			
$k = \frac{3}{4}$		A1cso			
Total			12		
(b)	May have “their” $y^2 - 14y + 49 = 0$ and attempt at formula or discriminant for first M1 with E1 awarded for being <i>completely correct</i> with both parts of conclusion such as “only one value of y / root/ solution so touches y -axis” or “discriminant = 0 therefore repeated root so touches y -axis” Second M1 can be earned for “their” $x^2 + 20x + 49 = 0$ and attempt at formula or discriminant with E1 awarded for being <i>completely correct</i> with both parts of conclusion such as “two (different) values of x / roots/ solutions so crosses x -axis in two places” May use geometry : award first M1 for stating x -coord of centre = “their” -10 and radius = “their” 10 ; then E1 for both stating that lengths are equal hence circle touches y -axis (or y -axis is a tangent to the circle) Second M1 for stating y -coord of centre = “their” 7 and radius = “their” 10 ; E1 for both explaining that $y_c < r$ and concluding statement that the circle crosses x -axis at two distinct points. To award each of these E1 marks all working must be correct.				
(c)(i)	Penalise trailing equals signs and any incorrect algebra even if recovered awarding A0cso All terms must be exactly as in the printed answer but “=0” may be on the LHS for A1cso. May use $x^2 + (kx+2)^2 + 20x - 14(kx+2) + 49 = 0$ and attempt to multiply out brackets for M1 giving $x^2 + k^2x^2 + 4kx + 4 + 20x - 14kx - 28 + 49 = 0$ leading to $(1+k^2)x^2 + 10(2-k)x + 25 = 0$ for A1cso				
(ii)	May write $10^2(2-k)^2 = 4 \times 25(1+k^2)$ for M1 then $4 - 4k + k^2 = 1 + k^2$ for A1 Correct discriminant appearing within the quadratic equation formula can earn M1.				

Q7	Solution	Mark	Total	Comment
(a) (i)	$x + x + (x + y) + (x + y) = 15$ OE	M1	2	ACF eg $2x + 2(x + y) = 15$
	$y = \frac{1}{2}(15 - 4x)$ OE	A1		
(ii)	$[S =] (x + y)^2 - y^2$ OE	M1	3	$[S =] x^2 + 2xy$ or $x(x + y) + xy$ etc Sub their y into correct S expression
	$= x^2 + x(15 - 4x)$	dM1		
	$= 15x - 3x^2$ $S = 3(5x - x^2)$	A1		
(b)(i)	$A - (x - 2.5)^2$	M1	2	must have both minus signs but A may be negative but not zero $p - (x - q)^2$; $p = 6.25, q = 2.5$ OE $\frac{25}{4} - \left(x - \frac{5}{2}\right)^2$
	$6.25 - (x - 2.5)^2$	A1		
(ii)	(Max S =) $3 \times$ "their" p (must be > 0)	M1	2	or $3(5 \times$ "their" q - "their" $q^2)$ OE eg $\frac{75}{4}$ withhold A1 if 2 marks not scored in (b)(i)
	$= 18.75$	A1		
Total			9	
(a)(i)	Penalise candidates who are clearly working back from printed answer in part (a)(ii)			
(ii)	Only allow dM1 if "their y" is of the form $a + bx$; expression need not be simplified so $S = (x + 7.5 - 2x)^2 - (7.5 - 2x)^2$, for example, earns dM1 . Must have "S = ..." for final A1 but allow $3(5x - x^2) = S$; condone "S m ² = ..." but not "S ² = ..." Allow omission of "S = ..." on final line provided it appears on an earlier line with "=" on all subsequent lines.			
(b)(i)	Example 1 $-(x - 2.5)^2 + \frac{25}{4}$ scores M1 A1 (and ISW any incorrect rearrangement)			
	Example 2 $-(x - 2.5)^2 - \frac{25}{16}$ scores M1 A0			
	Alternative for M1 : $x^2 - 5x = (x - 2.5)^2 - 2.5^2$ but must have both sides of this identity			
	If M0 is scored then award SC B1 for $6.25 - (x - 2.5)$ or $6.25 - (2.5 - x)^2$			
(ii)	Allow SC B1 if 18.75 is clearly obtained via differentiation or NMS			

Q8	Solution	Mark	Total	Comment
(a)	$\left(\frac{dh}{dt} =\right) 12t^2 - 59t + 72$ OE $t = 3 \Rightarrow \left(\frac{dh}{dt} =\right) 12 \times 3^2 - 59 \times 3 + 72$ $(108 - 177 + 72 =) 3$	M1 A1	4	two terms correct (may have x for t) all correct – must have t
(b)	(Decreasing) $\Rightarrow 12t^2 - 59t + 72 < 0$ $(4t - 9)(3t - 8)$ CVs are $(t =) \frac{9}{4}, (t =) \frac{8}{3}$ $\begin{array}{c} + \qquad \qquad - \qquad \qquad + \\ \hline \qquad \frac{9}{4} \qquad \qquad \qquad \frac{8}{3} \end{array}$ $\frac{9}{4} < t < \frac{8}{3}$ may have $t < \frac{8}{3}$ AND $t > \frac{9}{4}$	B1ft M1 A1 M1 A1		5
Total			9	
(b)	<p>For first M1, if using formula need to go as far as $\frac{59 \pm \sqrt{25}}{24}$ if inequality is correct and to a similar form if following through from their quadratic; if factorising quadratic (correct or incorrect) then factors should multiply out to give t^2 and constant terms to earn first M1.</p> <p>For second M1, if critical values are correct then sign diagram or sketch must be correct with correct CVs marked in correct order. However, if CVs are not correct then second M1 can be earned for attempt at sketch or sign diagram but their CVs MUST be marked correctly on the diagram or sketch. For final A1, inequality must have t and no other letter.</p> <p>If B1 is earned and correct quadratic inequality is seen, final answer of $\frac{9}{4} < t < \frac{8}{3}$ (with or without working) scores final 4 marks.</p> <p>(A) $\frac{9}{4} < x < \frac{8}{3}$ (B) $t < \frac{8}{3}$ OR $t > \frac{9}{4}$ (C) $t < \frac{8}{3}, t > \frac{9}{4}$ (D) $\frac{9}{4}, t, \frac{8}{3}$ (E) $\frac{54}{24} < t < \frac{64}{24}$ OE with or without working, each score 3 marks (SC3)</p> <p>If B1 is NOT earned, then only the next 3 marks are available, namely M1, A1, M1 and A0 even if final inequality is correct.</p> <p>Example NMS $t < \frac{8}{3}, t < \frac{9}{4}$ scores M1 A1 M0 (since both CVs are correct)</p>			